Calculus Summer Math 2022-2023 Name:_____

This practice is divided into four sections:

1. Simplify expressions
2. Evaluate functions
3. Solve equations
4. Basic functions and operations

It is crucial that you <u>do not use a calculator</u> for any of these questions.

1. Simplify each expression

a. $\dfrac{x^2 - 4x}{x^2 - 7x + 12}$

b. $\dfrac{5}{x} - \dfrac{2}{x}$

c. $\dfrac{x^3}{x^{-5}}$

d. $\dfrac{4 - x}{x^2 - 16}$

e. $\dfrac{x}{\sqrt{x}}$

f. $\dfrac{2}{\sqrt{3}}$

g. $\dfrac{a^{-1}}{a^{-2}\sqrt{a}}$

h. $\tan x \cos x$

i. $\sin^2 x + \cos^2 x$

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2. Evaluate each quantity and write in simplest form

Fill in this chart with the missing radian/degree measure:

Radians	$\frac{\pi}{6}$		$\frac{2\pi}{5}$				$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{4\pi}{3}$
Degrees		240°		180°	120°	225°			

Radians	$\frac{\pi}{4}$		$\frac{7\pi}{3}$				$\frac{5\pi}{4}$	$\frac{5\pi}{12}$	$\frac{11\pi}{6}$
Degrees		135°		90°	270°	300°			

Find the <u>exact value</u> of the following

$sin\left(\frac{\pi}{2}\right)$ $cos\left(\frac{\pi}{2}\right)$ $tan\left(\frac{\pi}{2}\right)$

$sin\left(\frac{\pi}{3}\right)$ $cos\left(\frac{\pi}{6}\right)$ $tan\left(-\frac{3\pi}{4}\right)$

$sin\left(\frac{4\pi}{3}\right)$ $cot\left(\frac{\pi}{3}\right)$ $tan\left(\frac{2\pi}{3}\right)$

$cos\left(-\frac{\pi}{3}\right)$ $cos\left(\frac{7\pi}{4}\right)$ $sin\left(\frac{5\pi}{2}\right)$

$ln\, e^7$ $ln\, 1$ $27^{\frac{2}{3}}$

$\sqrt{\frac{49}{121}}$ $\sqrt[3]{8}$ $16^{-\frac{1}{2}}$

e^0

3. Solve each equation/inequality for *x* (give exact values only; solve trig equations on $[0, 2\pi]$)

a. $3x + 2 = 8$

b. $4(x - 2) + 3x = -1$

c. $x^2 - 3x + 2 = 0$

d. $x^2 - 6x + 9 = 0$

e. $x^2 - 2x = 0$

f. $x^2 + 9 = 0$

g. $\frac{1}{x} + x = 4$

h. $\frac{2}{x + 1} = \frac{x - 2}{2}$

i. $\sqrt{x - 1} - 5 = 0$

j. $3cosx - 1 = 2$

k. $\frac{1}{3} = 3^{2x + 2}$

l. $5^{x + 1} = 25$

m. $ln\left(e^x\right) = 4$

n. $ln(x + 1) = 2$

o. $e^{2x - 5} = 1$

p. $lnx + 2lnx = 0$

q. $\sqrt{x + 3} = x - 9$

r. $\log_3 x = \log_3 4 - \log_3 5$

s. $(x + 3)(x - 3) > 0$

t. $\frac{x^4 - 1}{x^3} = 0$

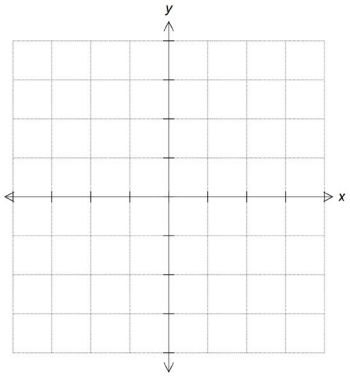
u. $\frac{x+2}{x} \geq 0$

4. Basic functions: graphs and operations

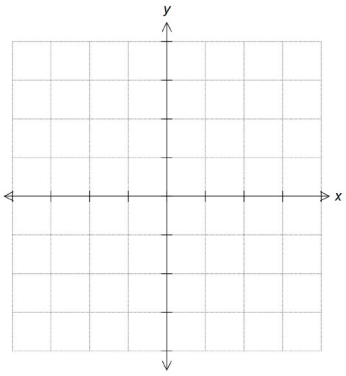
Calculus is a lot easier when you are familiar with several basic functions.
For each function listed below, you should be able to quickly and accurately:

- sketch its graph
- identify domain, range, intercepts, asymptotes…
- perform basic transformations (shifts, stretches, reflections)

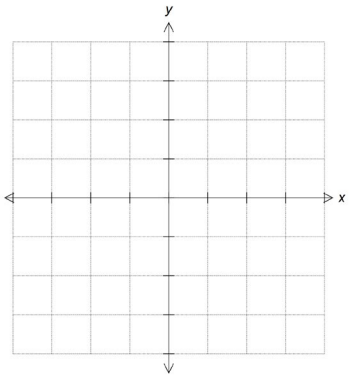
a. $y = x^2$



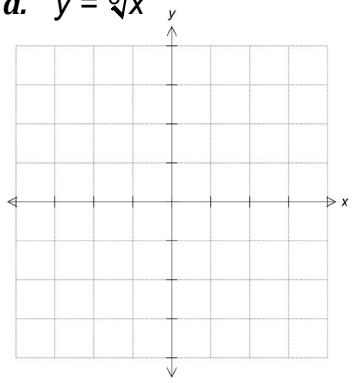
b. $y = x^3$



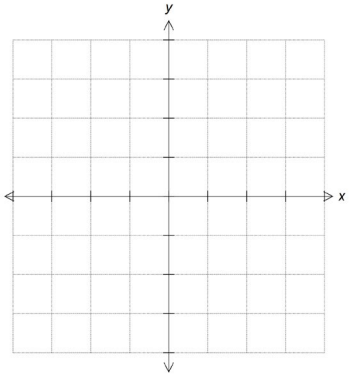
c. $y = \sqrt{x}$



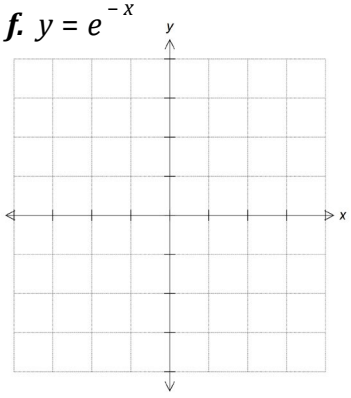
d. $y = \sqrt[3]{x}$



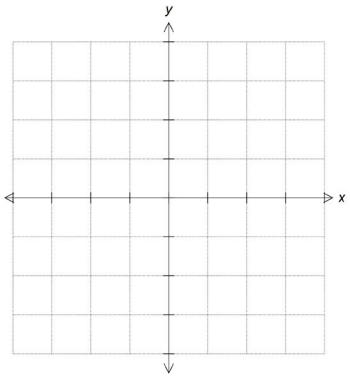
e. $y = e^x$



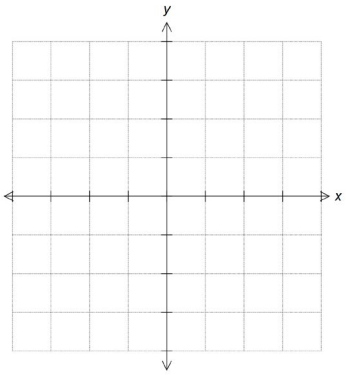
f. $y = e^{-x}$



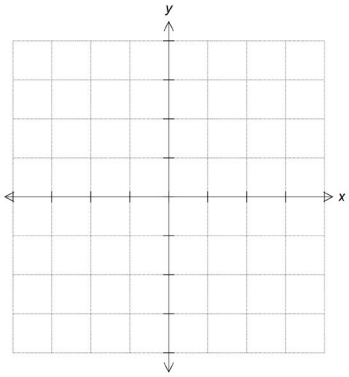
g. $y = \ln x$



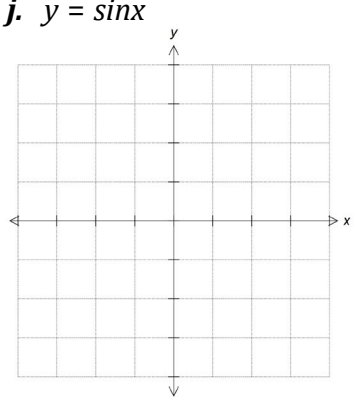
h. $y = \frac{1}{x}$



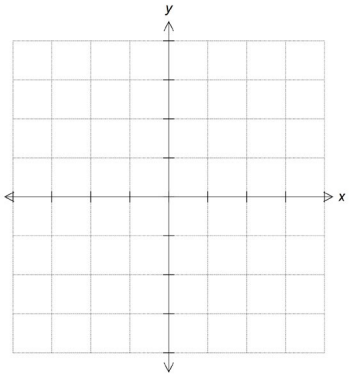
i. $y = |x|$



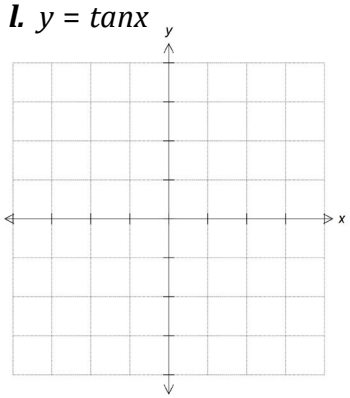
j. $y = \sin x$



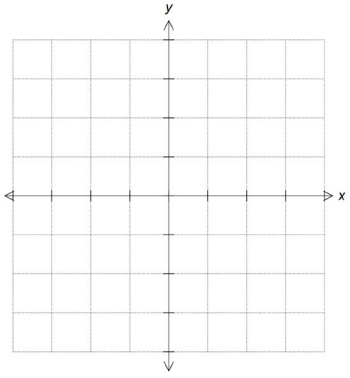
k. $y = \cos x$



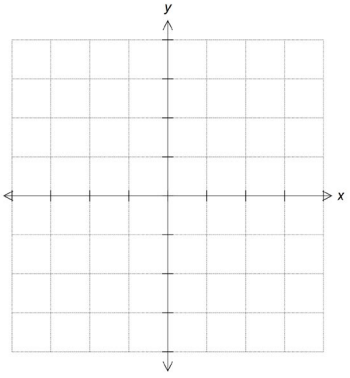
l. $y = \tan x$



m. $y = \begin{cases} \sin x & x < 0 \\ x^2 & x \geq 0 \end{cases}$



n. $y = \begin{cases} x + 1 & x < -1 \\ 2 - x & -1 < x \end{cases}$



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Operations with functions

o. Find the inverse of each function. Is $f^{-1}(x)$ a function? Why? Why not?

$f(x) = \sqrt{x+1}$ $f(x) = 5x - 2$ $f(x) = x^2 + 1$

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p. Given $f(x) = \sqrt{x+1}$ *and* $g(x) = 3x + 1$ evaluate and simplify the following:

$g(a + b) =$

$f(x - 3) =$

$f(g(x)) =$

$g(f(x)) =$

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q. Linear functions: find the equation of a line given each condition

- The line with slope 2 passing through (-3,1)
- The line parallel to $3x - y + 1 = 0$ and passing through the origin
- The line that forms a 45° angle with the x-axis and passes through (1,2)
- The line that is perpendicular to $y = \frac{1}{4}x + 1$ and is passing through (3,0)
- The line passing through points (-1,1) and (3,-1)
- The line with slope 2 that forms a triangle of area 12 with the positive x- and y-axis

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r. Find the domain of each function

$y = \sqrt{x+5}$ $y = \frac{3x}{x-1}$ $y = 2^x$

$y = ln(x + 2)$ $y = tan(x)$ $y = cos(\pi x)$

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s. Find the equation of each vertical and horizontal asymptote (if any)

$f(x) = \frac{x + 2}{2x - 1}$

$f(x) = \frac{3x}{x^2 + x - 2}$

$f(x) = \frac{x^3 + 5x^2}{x^2 + 9}$

$f(x) = \frac{x + 4}{x^3 - 4x}$

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t. Describe (and graph) each transformation when compared to the parent graph:

$y = \sqrt{x - 2} + 3$

$y = sin(2x)$

$y = |x^2 - 4|$

$y = \frac{1}{x + 3}$

$y = (x - 5)^2 - 2$

$y = ln(-x)$

---------- **The End** ----------

This packet is not for a grade. This will help you prepare for Calculus. Below are additional videos to help you work through the problems. Feel free to send me emails over the summer if you have questions are any parts

Unit Circle: https://www.youtube.com/watch?v=YK7KoU6ELWg

Trig Identities: https://www.youtube.com/watch?v=a70-dYvDJZY

Euler's Number (e) and Natural Log Properties: https://www.youtube.com/watch?v=daUlTsnCNRQ

Logarithmic Rules: https://www.youtube.com/watch?v=bPLyCH1WaEY